Fórmulas Cálculo IV

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1 P1

1.1 Fator Integrante

$$P(t).y'(t) + Q(t).y(t) = R(t)$$

$$y(t) = \frac{1}{\mu(t)} \cdot \int \frac{R(t)}{Q(t)} \cdot \mu(t).dt$$

$$\text{onde } \mu(t) = e^{\int \frac{Q(t)}{P(t)} \cdot dt}$$

1.2 "Método dos lambdas"

- Equações Diferenciais:

$$A.y"(t) + B.y'(t) + C.y(t) = x(t) \Rightarrow A.\lambda^2 + B.\lambda + C = 0$$

$$y_h(t) = C_1.e^{\lambda_1 t} + C_2.e^{\lambda_2 t}$$

$$y_p(t) \sim x(t) \rightarrow \text{Ex.: } x(t) = \sin(t) \Rightarrow y_p(t) = a.\sin(t) + b.\cos(t)$$

$$y(t) = y_h(t) + y_p(t)$$

- Equações de Diferenças:

$$A.y_{n+2} + B.y_{n+1} + C.y_n = x_n \Rightarrow A.\lambda^2 + B.\lambda + C = 0$$

$$y_{h_n} = C_1.\lambda_1^n + C_2.\lambda_2^n$$

$$y_{p_n} \sim x_n \rightarrow \text{Ex.: } x(t) = 2n^2 \Rightarrow y_{p_n} = a.n^2 + b.n + c$$

$$y(t) = y_{h_n} + y_{p_n}$$

2 P2

2.1 Sistemas de Equações

- Equações Diferenciais:

$$Y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \qquad Y'(t) = A.Y(t) + X(t)$$

Outra forma de se escrever o mesmo sistema:

$$y_1'(t) = a_{11}.y_1(t) + a_{12}.y_2(t) + x_1(t)$$

$$y_2'(t) = a_{21}.y_1(t) + a_{22}.y_2(t) + x_2(t)$$

$$Y_h(t) = e^{At} \cdot \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$e^{At} = \alpha(t).A + \beta(t).I$$

Sejam λ_1 e λ_2 os autovalores de A:

$$\lambda_1 \neq \lambda_2 : \quad \begin{array}{c} e^{\lambda_1 t} = \alpha(t).\lambda_1 + \beta(t) \\ e^{\lambda_2 t} = \alpha(t).\lambda_2 + \beta(t) \end{array}$$

$$\lambda_1 = \lambda_2 : \quad \begin{array}{c} e^{\lambda t} = \alpha(t).\lambda + \beta(t) \\ t.e^{\lambda t} = \alpha(t) \end{array}$$

$$Y_p(t) \sim X(t) \ \rightarrow \text{Ex: } X(t) = \begin{pmatrix} e^t + 2.e^{-t} \\ e^{-t} \end{pmatrix} \ \Rightarrow \ Y_p(t) = \begin{pmatrix} a.e^t + b.e^{-t} \\ c.e^{-t} \end{pmatrix}$$

$$Y(t) = Y_h(t) + Y_p(t)$$

* Sejam v_1 e v_2 autovetores de A:
$Y_h(t) = C_1.e^{\lambda_1 t}.v_1 + C_2.e^{\lambda_2 t}.v_2 \quad (\lambda_1 \neq \lambda_2)$
$Y_h(t) = C_1.e^{\lambda t}.v_1 + C_2.t.e^{\lambda t}.v_2 \quad (\lambda_1 = \lambda_2)$

- Equações de Diferenças:

$$Y_n = \begin{pmatrix} y_{1_n} \\ y_{2_n} \end{pmatrix} \qquad Y_{n+1} = A.Y_n + X_n$$

Outra forma de se escrever o mesmo sistema:

$$y_{1_{n+1}} = a_{11}.y_{1_n} + a_{12}.y_{2_n} + x_{1_n}$$

$$y_{2_{n+1}} = a_{21}.y_{1_n} + a_{22}.y_{2_n} + x_{2_n}$$

$$Y_{h_n} = A^n \cdot \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$A^n = \alpha_n.A + \beta_n.I$$

Sejam λ_1 e λ_2 os autovalores de A:

$$\lambda_1 \neq \lambda_2 : \quad \begin{matrix} \lambda_1^n = \alpha_n.\lambda_1 + \beta_n \\ \lambda_2^n = \alpha_n.\lambda_2 + \beta_n \end{matrix}$$

$$\lambda_1 = \lambda_2 : \quad \begin{matrix} \lambda^n = \alpha_n.\lambda + \beta_n \\ n.\lambda^{n-1} = \alpha_n \end{matrix}$$

$$Y_{p_n} \sim X_n \rightarrow \text{Ex: } X_n = \begin{pmatrix} n^2 \\ 2n \end{pmatrix} \Rightarrow Y_{p_n} = \begin{pmatrix} a.n^2 + b.n + c \\ d.n^2 + e.n + f \end{pmatrix}$$

$$Y_n = Y_{h_n} + Y_{p_n}$$

* Sejam v_1 e v_2 autovetores de A:
$Y_{h_n} = C_1.\lambda_1^n.v_1 + C_2.\lambda_2^n.v_2 \quad (\lambda_1 \neq \lambda_2)$
$Y_h(t) = C_1.\lambda^n.v_1 + C_2.n.\lambda^{n-1}.v_2 \quad (\lambda_1 = \lambda_2)$

3 P3

3.1 Série de Taylor

$$f(x) = f(a) + f'(a).(x-a) + \frac{f"(a)}{2!} \cdot (x-a)^2 + \frac{f'''(a)}{3!} \cdot (x-a)^3 + \ldots$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} \cdot (x-a)^n$$

3.2 Séries de Taylor "manjadas"

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots = \sum \frac{x^n}{n!}$
$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots = \sum (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}$
$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots = \sum (-1)^n \cdot \frac{x^{2n}}{(2n)!}$
$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots = \sum (-1)^n \cdot \frac{x^{n+1}}{n+1}$ (para $|x| < 1$)
$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \cdots = \sum x^n$ (para $|x| < 1$)

4 Observações

$$\sinh(x) = \sin(ix) = \frac{e^x - e^{-x}}{2}$$
$$\cosh(x) = \cos(ix) = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2(x) - \sinh^2(x) = 1$$
$$e^{ix} = \cos(x) + i.\sin(x)$$